Choosing Channel Quantization Levels and Viterbi Decoding for Space Diversity Reception Over the Additive White Gaussian Noise Channel

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In this article we review previous work in the area of choosing channel quantization levels for a additive white gaussian noise channel composed of one receiver-demodulator, and show how this applies to the Deep Space Network composed of several receiver-demodulators (space diversity reception). Viterbi decoding for the resulting quantized channel is discussed.

I. Introduction

The Deep Space Network receives interplanetary signals at several locations on the Earth. Also, at each complex there are several receiver-demodulators. Thus, we have available a number of receiver-demodulators, each producing a "stream" of sufficient statistics (symbols). For biphase modulation over the unquantized additive white gaussian noise channel (AWGN), it has been shown that maximum likelihood decoding can be realized by first forming a weighted sum of these streams to form a new single stream of statistics, followed by maximum likelihood decoding of this single stream (Ref. 1). This is commonly referred to as symbol stream combining. Each stream of statistics is weighted in proportion to the square root of its received signal energy divided by the noise power spectral density associated with its corresponding receiver-demodulator.

In this article, we are concerned with the quantized AWGN channel, and so the above does not apply. How are the quantization levels for each receiver-demodulator to be chosen and what is the structure of the Viterbi decoder? Choosing quantization levels for the AWGN channel composed of one receiver-demodulator is not new (Ref. 2). This will be reviewed and extended for space diversity reception, resulting in an optimum method for the Deep Space Network. Furthermore, we discuss the Viterbi decoder for space diversity reception.

II. Review of Previous Work

It has been proposed (Refs. 2 and 3) that modulation and demodulation design can be based upon the "R_o criterion," where R_o is the cut-off rate of the channel. That is, modulation and demodulation should be such that the cut-off rate of the resulting channel created by the modulation and demodulation scheme is maximum. The rationale for this is the fact that the probability of bit error P_b for the best convolutional codes is upper bounded by

$$P_b \leqslant \frac{2^{-KR_o/R}}{[1 - 2^{-\epsilon R_o/R}]^2} \tag{1}$$

where $\epsilon > 0$ and $0 \leqslant R \leqslant R_o(1 - \epsilon)$, where R is the code rate in bits/channel-symbols and K is the constraint length (Ref. 4).

In our case, we are interested in choosing the quantization levels such that R_o is maximum.

The cut-off rate in bits/channel-symbols is

$$R_o = -\log_2 \left\{ \min_{q(x)} \sum_y \left[\sum_x \sqrt{p(y|x)}\, q(x) \right]^2 \right\} \quad (2)$$

where x is the transmitted symbol, $q(x)$ is a probability distribution, and $p(y|x)$ is the conditional probability distribution of the received output symbol y given that the symbol x is transmitted. We are interested in biphase modulation, where x is one of two symbols, say $x = \pm\sqrt{E_s}$ (E_s is the received symbol energy). Then for biphase modulation, we have by symmetry that $q(x = +\sqrt{E_s}) = q(x = -\sqrt{E_s}) = 1/2$ is the distribution needed to minimize the bracketed expression in the definition of R_o, Eq. (2). Thus, we have

$$R_o = 1 - \log_2 \left[1 + \sum_y \sqrt{p(y|x = +\sqrt{E_s})\, p(y|x = -\sqrt{E_s})} \right] \quad (3)$$

Let us consider the symmetric quantization scheme of Fig. 1, where Q, the size of the output alphabet, is assumed to be even. In this article, we shall only consider the case of Q being even. The case of Q being odd can be handled in essentially the same way. Thus, for symmetric channels, we have $T_i = -T_{-i}$, $i = 1, 2, \ldots, Q/2$. By convention, we shall define $T_o = 0$ and $T_{Q/2} = -T_{-Q/2} = \infty$.

In Fig. 1, we have denoted the output alphabet by $\{a_i, i = -Q/2, \ldots, -1, 1, \ldots, Q/2\}$. Thus, the quantization scheme of Fig. 1 produces the discrete input-binary, output-symmetric channel of Fig. 2 with transition probability distributions $p(a_i|x)$ given by

$$p(a_i|x = \pm\sqrt{E_s}) = \begin{cases} \dfrac{1}{\sqrt{\pi N_o}} \displaystyle\int_{T_{i-1}}^{T_i} e^{-(z \pm \sqrt{E_s})^2/N_o} dz, & i = 1,2,\ldots,Q/2 \\[2ex] \dfrac{1}{\sqrt{\pi N_o}} \displaystyle\int_{-T_{-i}}^{-T_{-i-1}} e^{-(z \pm \sqrt{E_s})^2/N_o} dz, & i = -1,-2,\ldots,-Q/2 \end{cases} \quad (4a)$$

where N_o is the one sided noise spectral density. In terms of the normalized quantization $\hat{T}_i \equiv T_i/\sqrt{E_s}$ and signal-to-noise ratio $\rho \equiv E_s/N_o$, the above transition probability can be expressed as

$$p(a_i|x = \pm\sqrt{E_s}) = \begin{cases} \sqrt{\rho/\pi} \displaystyle\int_{\hat{T}_{i-1}}^{\hat{T}_i} e^{-(z\pm 1)^2 \rho}\, dz & i = 1, 2, \ldots, Q/2 \\[2ex] \sqrt{\rho/\pi} \displaystyle\int_{-\hat{T}_{-i}}^{-\hat{T}_{-i-1}} e^{-(z\pm 1)^2 \rho}\, dz & i = -1, -2, \ldots, -Q/2 \end{cases} \quad (4b)$$

Using Eq. (4a) with Eq. (3), and setting $\partial R_o/\partial T_i = 0$, $i = 1, 2, \ldots, Q/2$, we have after some algebra the set of equations:

$$\cosh[2\hat{T}_i\rho - 1/2 \ln \Lambda(a_i)] = \cosh[2\hat{T}_i\rho - 1/2 \ln \Lambda(a_{i+1})] \qquad i = 0, 1, \ldots, Q/2 - 1 \quad (5)$$

where $\Lambda(a)$ is the likelihood ratio

$$\Lambda(a) \equiv \frac{p(a|x = \sqrt{E_s})}{p(a|x = -\sqrt{E_s})} \quad (6)$$

Notice that the equation $\cosh(x) = \cosh(y)$ can be satisfied by one of the two conditions: (1) $x = y$, or (2) $x = -y$. Numerical results show that the former condition applied to Eq. (6) cannot be satisfied for any T_i. Thus, we must use the second condition to obtain

$$8\hat{T}_i\rho = \ln[\Lambda(a_i)\Lambda(a_{i+1})] \qquad i = 0, 1, \ldots, Q/2-1 \quad (7)$$

It can easily be shown that Eq. (7) is equivalent to Eq. (16) of Ref. 2.

Massey (Ref. 2) suggested the following algorithm to solve Eq. (7): Pick some T_1 and use Eq. (7) with $i = 1$ to solve for T_2 (remember that $T_o = 0$). Likewise, use Eq. (7) with $i = 2$ to solve for T_3. Continuing in this way, we finally arrive at a tentative value for $T_{Q/2-1}$. Then, see if this value for $T_{Q/2-1}$ satisfies Eq. (7) for $i = Q/2 - 1$, remembering that $T_{Q/2} = \infty$. If not, then pick a smaller T_1 and try again. If this method still fails to converge, then the tentative guesses for T_1 should be increased instead of decreased.

The fact that the above algorithm may be numerically intensive is not important, since the normalized quantization levels can be determined off-line. Thus, a table look-up can be stored in ROM, giving the normalized quantization levels as a function of signal-to-noise ratio. Notice that the quantization levels are a function of both E_s and N_o. The maximum likelihood estimation of these parameters has been considered in Ref. 5.

III. Space Diversity Reception

For space diversity reception, we have multiple receiver-demodulators, say N of them. If $\mathscr{H}^{(i)}$ is the output space of the ith receiver-demodulator, then the output space $\mathscr{H}$ of the discrete channel composed of N receiver-demodulators is $\mathscr{H}^{(1)} \times \mathscr{H}^{(2)} \times \cdots \times \mathscr{H}^{(N)}$. Let $\mathscr{H}^{(i)}$ be the alphabet $\{a_1^{(i)}, a_2^{(i)}, \ldots, a_Q^{(i)}\}$ (without loss of generality, we assume the number of quantization levels for each receiver-demodulator to be the same). Assuming that the noise of the N receiver-demodulators are uncorrelated, we have that the joint transition probabilities of this discrete channel are

$$p_{\mathscr{H}}(\mathbf{a}|x) = \prod_{i=1}^{N} p_{\mathscr{H}^{(i)}}(a^{(i)}|x) \tag{8}$$

where $p_{\mathscr{H}^{(i)}}(a^{(i)}|x)$ are the marginal transition probabilities of the quantized channel comprised of just the ith receiver-demodulator, and

$$\mathbf{a} \in \mathscr{H}, \quad \mathbf{a} \equiv (a^{(1)}, a^{(2)}, \ldots, a^{(N)}), \quad a^{(i)} \in \mathscr{H}^{(i)} \tag{9}$$

The cutoff rate of the discrete channel composed of N receiver-demodulators is just

$$R_o = 1 - \log_2 \left[1 + \sum_{\mathbf{a} \in \mathscr{H}} \sqrt{p_{\mathscr{H}}(\mathbf{a}|x = \text{"0"})\, p_{\mathscr{H}}(a|x = \text{"1"})} \right]$$

$$= 1 - \log_2 \left[1 + \prod_{i=1}^{N} \sum_{a^{(i)} \in \mathscr{H}^{(i)}} \sqrt{p_{\mathscr{H}^{(i)}}(a^{(i)}|x = \sqrt{E_s^{(i)}})\, p_{\mathscr{H}^{(i)}}(a^{(i)}|x = -\sqrt{E_s^{(i)}})} \right] \tag{10}$$

where $E_s^{(i)}$ is the received symbol energy at the ith receiver-demodulator.

From Eq. (10), we see that choosing the quantization levels to maximize R_o for the channel with N receiver-demodulators is equivalent to choosing the quantization levels of each receiver-demodulator to maximize its corresponding cut-off rate.

IV. Viterbi Decoding

For a branch in the trellis corresponding to the transmitted symbol sequence $(x_1, x_2, \ldots, x_n)$, where we have a rate $1/n$ code, we must calculate the metric

$$\sum_{j=1}^{n} \ln p_{\mathscr{H}}(\mathbf{a}_j|x_j) \tag{11}$$

where $\mathbf{a}_j, j = 1, \ldots, n$ is the received symbol sequence composed of symbols from $\mathscr{H}$. From Eq. (8), we see that the metric of Eq. (11) is just

$$\sum_{j=1}^{n} \sum_{i=1}^{N} \ln p_{\mathscr{H}^{(i)}}(a_j^{(i)}|x_j) = \sum_{i=1}^{N} \sum_{j=1}^{n} \ln p_{\mathscr{H}^{(i)}}(a_j^{(i)}|x_j) \tag{12}$$

Thus, we see from Eq. (12) that Viterbi decoding is accomplished by calculating the metrics of each branch in the trellis associated with each receiver-demodulator, and summing these metrics to give the total metric associated with that branch.

V. Remarks

This article reviews how quantization levels should be chosen to maximize the cut-off rate of the discrete channel created by a quantization scheme. For most situations, eight properly chosen quantization levels will give nearly optimum results, (Refs. 2 and 4). However, since a coding gain of only one tenth of a decibel in received bit energy-to-noise ratio is important to the Deep Space Network, more quantization levels may be required.

We have also discussed the Viterbi decoder for space diversity reception over the quantized channel. In a sense, the "combining" of the symbol streams is performed "within" the Viterbi decoder, where the branch metrics are given by the sum of metrics associated with each receiver-demodulator, Eq. (12). It is therefore not optimum to combine the symbols before Viterbi decoding. An example of a suboptimum method is to treat the symbols as real numbers (perhaps the midpoint of the symbol's corresponding quantization zone), and combine them as explained in Ref. 1.

It should be noted that symbol stream combining has been carried out experimentally in Ref. 6, and practical aspects of choosing quantization levels have been discussed in Ref. 7. However, the "R_o criterion" was not considered in these references, nor were the symbols "combined" as discussed here. Does choosing the quantization levels to maximize R_o and performing "true" maximum likelihood (Viterbi) decoding (Eq. 12) lead to any significant coding gain? This question should be investigated further to make sure that the Deep Space Network is making the best use of the channel.

Acknowledgment

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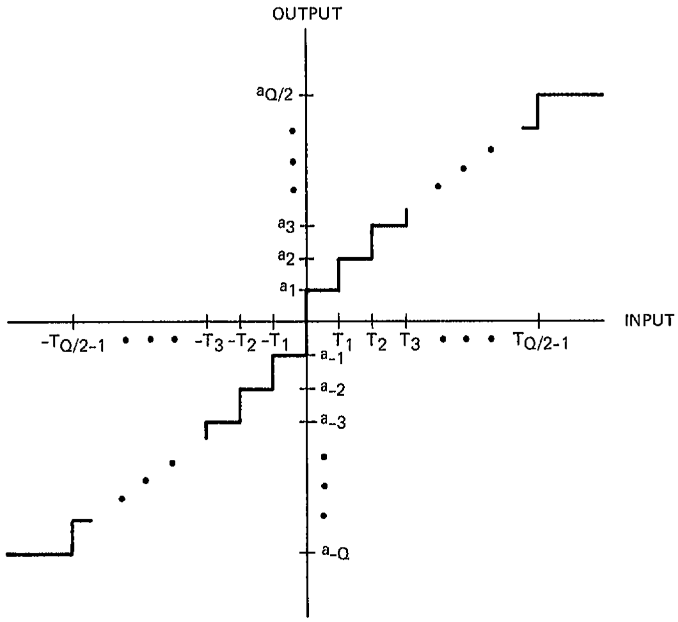

Fig. 1. Quantization scheme

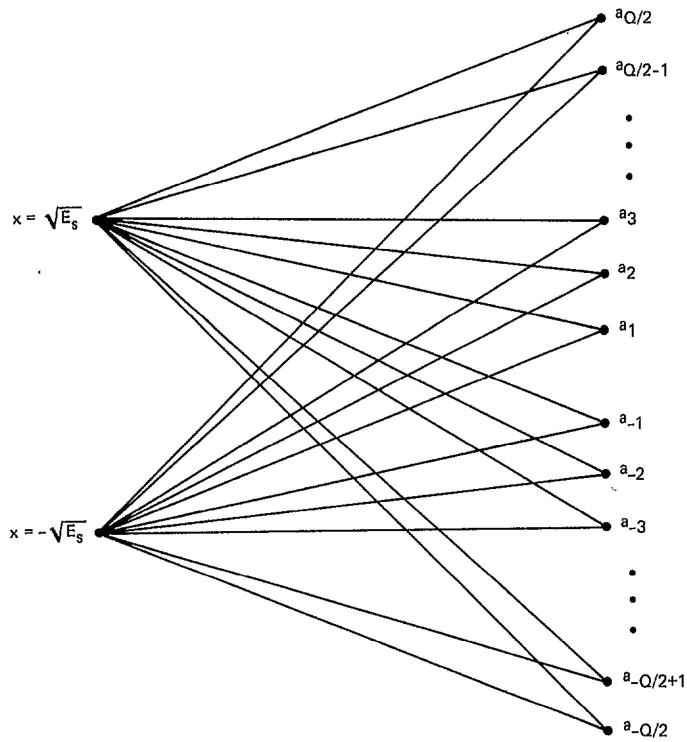

Fig. 2. Discrete channel